Storytelling and Mathematics

Why? Why would we tell stories in math?

Mathematics teachers are usually consumed with teaching algorithms. Algorithms, however, are based on axioms and those can vary depending on the theory of mathematics. Storytelling in mathematics permits students to explore deductive reasoning.

In this session, you'll have the opportunity to explore deductive reasoning, modeling, and axiomatic systems through the use of storytelling. The explorations will be conducted with Project Zero thinking routines, Socratic Seminars on primary sources, and inquiry-based learning. We'll also explore exemplar work from previous students already published on iTunes Store.

What will you get from this training?

1. Strategies to help you develop Mathematical Stories by reasoning deductively about historical themes and concepts from mathematical axioms and axiomatic systems.

2. Creating opportunities for students to build connections with what they already know.

3. Explore mathematical and historical concepts from many different angles, employ divergent thinking, and practice the act of thinking itself through developing Story.

<u>Launch Activity</u> - Pechakucha Overview of the Project

A visual and guided tour of some of the student exemplars and the process using the *Pechakucha* strategy, i.e., 20 seconds and 20 slides. There are a limited amount of exemplars available at each table so participants are encouraged to download the student iBooks by navigating to the iBooks Store, and searching for Santa Fe High School Innovation Academy. (iOS devices not supported at present.) There is a survey link at the end of this resource and we will provide participants with digital copies of every resource.

It all began with Touchstones and my excellent coffee and John pushing me to create a math project that would reach an authentic audience. Have students create math stories is a cross-curricular unit that takes social studies concepts about power, justice, freedom, etc., and fuses them together with mathematical content. The students that wrote these stories were enrolled in Geometry or Algebra classes with Mr. Haack and were simultaneously enrolled in

NM History with Mr. Morrison. The students wrote short stories with a buddy, used iBooks author, and published their completed works to the Apple Store. Open iBooks and search for "Santa Fe High School Innovation Academy" and you can download the books. The unit took the better part of a year, and the process helped the students and the teachers think more creatively and effectively about history, math, and society. We were trying to build a project that would use the technology available at the high school and would reach a wider and authentic audience. We thought we would be creating a history of math, or the ability for students to connect math with physics, but in the end we found that storytelling was a more meaningful way for students to think, and think about math itself, and to develop a deeper understanding of the issues in their lives.

<u>Exploration Activity</u> - Creating Your Story

The leader begins with an open ended math and social studies question, e.g., ***Is a fraction free? Are parallel lines fair?*** Then, consider with your groups, "What are the axioms that guide this freedom? That guide this fairness? How might you model and describe this freedom, fairness, etc., in a discussion by reasoning deductively from your axioms?"

<u>Practice Activity</u> - Using Mathematical Inquiry Frames as a Thinking Routine [We Do This Together]

Progress [We Do This Together]

- ***How does a circle get better?*** Circles get stronger by getting bigger. Circles acquire more circumference by subsuming lines.
- ***What is progress to a circle?*** Circles are perfect polygons. Polygons are imperfect circles. A circle represents the progression of a polygon to perfection, or the limit as the sides and angles become infinitely small. Alternately, there is no circle, there are only many-sided polygons.

Justice [We Do This Together]

- ***What is injustice to a sequence?*** A sequence that does not converge on a sum is imperfect and irrational. Sequences are infinite or finite. Sequences are cousins to series. Series are the ruling class and sequences are the proletariat class.
- ***What does a sequence think is fair?*** Do I get to end or do I go on forever? This suffering never ends … I hate being an infinite series. Or, I am finite … how do I become an infinite series?

<u>Remembrance Activity</u> - Presenting 'Elevator Pitches' & Cooperative Planning Notes

Poster Paper or Gdoc: After the modeling of the math inquiry frames as a thinking routine, please discuss the remaining themes and concepts below in your small groups. Your goal is to create a poster paper or Google Doc and share it with jhaack@sfps.k12.nm.us and address the following:

- What is the social commentary in your story? What themes and concepts did you utilize?
- How might leveraging mathematics and axioms make the purpose more effective than otherwise?
- How did the thinking routines help you tease out your ideas?

1-Minute Elevator Pitch: Please delegate one person from your small group to deliver a one minute elevator pitch on your proposal to the workshop audience. These pitches will be recorded and shared with you at the end of the training.

Mathematical Inquiry Frames - Crafting a Mathematical Story [Finish These on Your Own]

We have provided examples and starters below. Remember - these are examples. We encourage you to develop your own axioms, your own theme, characterization, climatic event. Practicing this on your own today will help you model this process with your students. You do not need to go through every concept, only the ones that you find are necessary to develop your social commentary within your Mathematical Story.

Power [Finish These on Your Own]

- ***What is power for shapes?*** How does one shape win the hearts and minds of all other shapes? Polygons below the six sided hexagon cannot serve in government. Circles rare bullies and triangles are non-violent.
- ***What is hegemonic power for a triangle?*** Equilateral triangles have an intellectual edge over the isosceles triangle because of their inherent perfection. Angle-Side-Angle triangles usurp the power of Side-Side-Angle triangles who might not even exist.

Freedom [Finish These on Your Own]

- ***What does it mean for shapes to be free?*** The coordinate plane is free because it approaches infinity. Regular polygons are free but convex polygons are slaves. Regular polygons are conformists.

Order [Finish These on Your Own]

- ***How are shapes, sequences, and or equations predictable?*** Functions possess order because every input has exactly one output, but relations have no purpose.
- ***Are coordinate planes infinite?*** The universe is finite. The universe is infinite.

In addition to historical themes, you and your cooperative groups can consider using the concept list below to begin your development of your axioms, characterization, rising event, etc. These may be used in conjunction with or as alternates to the themes above. Practicing these now as you develop your elevator pitch will help you model these later for your students.

<u>List of Concepts</u> - What historical concept are you teaching with your Mathematical Story?

- **Progress**, i.e., what is progress to a fraction [or other math concept]?
- **Justice**, i.e., what is justice to a fraction [or other math concept]?
- **Conflict**, i.e., what is conflict to a fraction [or other math concept]?
- **Freedom**, i.e., what is freedom to a fraction [or other math concept]?
- **Order**, i.e., what is order to a fraction [or other math concept]?
- **Power**, i.e., what is power to a fraction [or other math concept]?

<u>Themes</u> - How does your Mathematical Story reflect social and historical themes?

Power; the ability to get your way over the objections of others.

- Soft Power: ideas and actions - Hearts and Minds
- Hard Power: Physical
- Hegemonic Power: the ability to make the “rules of the game”
- Economic Power: the ability to deliver value to participants
- Freedom/Rights
- Progress/Growth
- Stability/Predictability
- Change/Continuity
- Interpretation
- Motivation/Self-Interest
- Conflict
- Compromise
- Deviance
- Nostalgia

History; the perennial themes of social Discourse.

- Arts generally reflect society.

- Different places become connected and dependent through communication of ideas, trade, and sharing of resources.
- Individuals use their own values and perspectives to interpret history.
- A democratic society encourages, but does not guarantee equality of opportunity and equality before the law.
- The more complex a society becomes, the greater is the need for effective leadership and positive human interaction.
- Through governments, organizations, and institutions people modify and regulate the economy to achieve the goals of :
    - economic justice,
    - stability,
    - freedom, and
    - growth.
- A period of time normally exists between the recognition of a problem and its ultimate resolution.
- Power can be used to achieve both constructive and destructive ends.
- Conflict unresolved by compromise and/or change may lead to violence.

Justice & Fairness; Political implementation of concepts and themes in social space.

- Who decides?
- Who enforces?
- Who determines right? Who determines wrong?
- Economic
- Social
- Cultural

Every kid brings an understanding of these concepts. However brilliant, simplistic, or possibly even misinformed, every student has a concept of what justice, progress, conflict, freedom, order, and power mean. In that sense, the more sophisticated the understanding, the more sophisticated the story. The students differentiate the project themselves. Every student has the ability to write this story, because they believe they know what is right, fair, just, etc.

Thinking Routines

Another way to encourage thinking about this process with your students is to use the Thinking Routines from Harvard's Visible Thinking Network. When we first developed and conducted this with our students we were not aware of this resource. Below, we have listed two example routines that might help your students think meaningfully about the value of the math storytelling and provide students a way to connect this creative process with the explicit

content and standards they must also learn. More information on these routines can be found here: http://www.visiblethinkingpz.org/ We are very grateful to have found this new resource and look forward to the second round of stories that will utilize these routines. As with any project, one of the most rewarding aspects is watching how it evolves and grows through its phases of implementation.

Thinking Routine; The 3 Why's

- Why might creating a story [about, e.g., a fraction] matter to us (students)?
- Why might creating a story [about, e.g., 2D geometry] matter to people around us?
- Why might creating a story [about, e.g., trigonometry] matter to the world?

Consider using this routine in the early stages of story development. This might help students get the larger picture of how the story serves as a social commentary, and provide them question frames to help them brainstorm how using mathematics, axioms, and deductive reasoning will enrich the commentary for a larger authentic audience. Consider using this routine today if you struggle with developing themes and concepts.

Thinking Routine; Connect-Extend-Challenge

- Connect - How did the presentation about math storytelling *connect* with prior knowledge you had?
- Extend - What new ideas did you get from this training that pushed your ideas in new directions?
- Challenge - What is still challenging to you about the process of math storytelling?

Consider using this routine after delivering content within the regular course of instruction. This routine will help students connect common mathematical standards with advanced forms of mathematical reasoning. Later, when students enter late high school, or college, these concepts will be introduced together in Fibonacci sequences, the Golden Mean, Euler's identity, Quaternions, Lobachevskian or Non-Euclidean Geometry, etc. You will also use this routine today as an evaluation of our training, and considering these questions during your remembrance activity might help you develop your themes and concepts.

Question & Answer

Open forum for questions. Please do not ask follow up questions. You are encouraged to email us at jhaack@sfps.k12.nm.us and jmorrison@sfps.k12.nm.us.

Google Form & Evaluation

Please visit the following Google Form (http://goo.gl/forms/rXqrclvDdL) site to evaluate our training. Please provide your email so that we can share the digital resources with you later tonight. Thanks for attending our training and as always, we welcome your ideas and insight in the ongoing evolution of this project. As stated above, for this evaluation we used the Connect-Extend-Challenge thinking routine to get information about what you connected this training with, how you pushed your ideas, and what challenges or questions might still remain.

Addenda

Project Logistics

- How long did we take?
  - 3 Quarters
- What did we do?
  - Seminar on Euclid's Elements or Al-Kwarizmi (3 days)
  - Seminar on Flatland passages (3 days)
  - Viewing of Flatland the Movie (3 days)
  - Brainstorm Webs (2 days)
  - Free-Writes (1 for each source)
  - Rough Drafts (2-4 weeks of editing and revision
  - Google Drawing Tutorials (1 day)
  - Photoshop Tutorials (1 day)
  - Hand Drawing & Scanning Tutorials (1 day)
  - iBooks Author Tutorial (Ongoing)
  - Publish Week
  - Presentation Week
- When did we do it?
  - On block days, we used the last 15-45 minutes for the components above
  - Four days out of the entire time were used exclusively for project work
- How did the students' math results compare to the other students at the school?
  - The class composite scores were in all cases at or above every other section of Geometry as measured by DEA.

Historical Insights

"Writing is a concentrated form of thinking...a young writer sees that with words he can place himself more clearly into the world. Words on a page, that's all it takes to help him separate himself from the forces around him, streets and people and pressures and feelings. He learns to think about these things, to ride his own sentences into new perceptions." **Don DeLillo**

"No problem can withstand the assault of sustained thinking." **Voltaire**

$e^{i\pi}+1=0$ - **Euler**

Euclid's Elements Defintion 23: Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet on another in either direction. - **Euclid**

Euclid's Elements Postulate 5: That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. - **Euclid**

Seminar Questions for Euclid:

- Does Euclid's text suggest that parallel lines exist or does it only define them?
- Why does Postulate 5 not use the term parallel?
- Why is it that intersecting lines are a postulate while parallel lines are a definition?

The Compendious Book on Calculation by Completion and Balancing. A seminal text on how wealth is passed or exchanged under Islamic law that used mathematics as a framework ensuring equity. - **Al-Kwarizmi**

The Battle of the Books - **Jonathan Swift**

The Planiverse **https://www.goodreads.com/book/show/866402.The_Planiverse**

Flatland - *http://www.goodreads.com/book/show/433567.Flatland*

Spaceland https://www.goodreads.com/book/show/274052.Spaceland

Flatterland https://www.goodreads.com/book/show/17033.Flatterland

E - The Story of a Number - https://www.goodreads.com/book/show/271361.e

A Brief History of π - https://www.goodreads.com/book/show/586616.A_History_of_Pi

The Life of π - https://www.goodreads.com/book/show/19193315-the-life-of-pi

Flatland: The Movie (2006) - http://www.flatlandthemovie.com/

The Fabulous Fibonacci Numbers - http://www.goodreads.com/book/show/909093.The_Fabulous_Fibonacci_Numbers

The Golden Ratio: The Story of Phi, The World's Most Astonishing Number - http://www.goodreads.com/book/show/24081.The_Golden_Ratio

Mathematical Standards & Practices Addressed

There are numerous standards that could be addressed that vary on the content of the course you teach. Geometry is a good place to start because it is a subject that is present in every level of the CCSS. The students worked on this project for the entire year and drew from standards across the geometry curriculum. In the beginning of the high school Geometry domain, the authors of the CCSS call for:

> An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most ef cient use of material.
>
> Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)
>
> During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise notions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

This is precisely the type of the geometrical thinking and reasoning that creating a Mathematics Story fosters. That is, in developing their own axiomatic systems and reasoning from them, students are introduced to the concept of developing different geometries. This

allows students to break down one of the largest misconceptions about mathematics, i.e., that all of its rules are fixed and unchanging. The reality is that there are many different "maths" that are utilized within different tuples, planes, etc., some of which are real, some complex, etc., and that by encouraging students to think in this fashion from 6th through 9th grade, they are opened up to world of divergent thinking and mathematics that is all too often stifled in the secondary Discourse. These layers of mathematics will foster deep and rich thinking about the universe. Consider Euler's famous identity above, in which the imaginary, the irrational, the transcendental, the unit, and the zero, are all intertwined in one identity. $e^{i\pi}+1=0$ Exposing middle school and early high school students to the mysteries of math Discourse through Story will only deepen their later understanding of Calculus, Discrete Mathematics, Statistics, and Physics C, etc.

Additionally, the Mathematics Storytelling project utilized all of the Mathematical Practice Standards:

1. ***Make sense of problems and persevere in solving them***
2. ***Reason abstractly and quantitatively***
3. ***Construct viable arguments and critique the reasoning of others***
4. ***Model with mathematics***
5. ***Use appropriate tools strategically***
6. ***Attend to precision***
7. ***Look for and make use of structure***
8. ***Look for and express regularity in repeated reasoning***

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